(7 pages) **Reg. No. :** ....

Code No. : 30422 E **Sub. Code : CMMA 51**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics — Core

LINEAR ALGEBRA

(For those who joined in July 2021–2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $T: V \to W$ is defined by $T(v) = v$, for all $v \in V$, then its Kernel is ————

   (a) $\{0\}$

   (b) V

   (c) W

   (d) ϕ

2. In a vector space $\alpha v = 0$ implies ————

   (a) $\alpha = 0$ (b) $v = 0$

   (c) (a) or (b) (d) (a) and (b)

3. The set ———— is linearly independent in $V_2(R)$

   (a) $\{(1, 0), (0, 2)\}$ (b) $\{(1, 0), (2, 0)\}$

   (c) $\{(1, 0), (0, 0)\}$ (d) $\{(0, 0)\}$

4. Which of the following is true?

   (a) $L(S) \le S$ (b) $L(S) = S$

   (c) $S \le L(S)$ (d) $S \ne L(S)$

5. In $V_3 R$, if $x = (1, 2, 3)$, then $\|x\| =$ ————

   (a) $\sqrt{6}$ (b) 6

   (c) 14 (d) $\sqrt{14}$

6. If $T: V \to \frac{V}{W}$ is the natural homomorphism, then its rank is ————

   (a) dim V (b) dim W

   (c) dim V + dim W (d) dim V – dim W

7. The characteristic polynomial of the matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is ————

(a) 0 (b) x

(c) $2x$ (d) x^2

8. If the matrices A and B are similar, then ————

(a) $|A| = |B|$ (b) $|A| \neq |B|$

(c) $|A| = |B|^2$ (d) $|A|^2 = |B|$

9. The product of the characteristic roots of the matrix $\begin{pmatrix} 1 & -1 \\ -3 & 3 \end{pmatrix}$ is ————

(a) 1 (b) 6

(c) -6 (d) 0

10. If A is an orthogonal matrix, then which of the following can be a characteristic root?

(a) -1 (b) 0

(c) 2 (d) -2

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Let V be a vector space over F. Derive a necessary and sufficient condition for a non empty subset W of V to be a subspace of V.

Or

(b) Prove that $T: R^2 \to R^2$ defined by $T(a, b) = (2a - 3b, a + 4b)$ is a linear transformation

12. (a) Let V be a finite dimensional vector space over a field F. Prove that any linearly independent set of vectors in V is a part of a basis.

Or

(b) Let V be a vector space over a field F. Let $S, T \leq V$. Prove :

(i) $S \leq T \Rightarrow L(S) \leq L(T)$

(ii) $L(S \cup T) = L(S) + L(T)$.

13. (a) State and prove the Triangle inequality.

Or

(b) Obtain the linear transformation $T: V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ with respect to the standard basis $\{e_1, e_2, e_3\}$.

14. (a) Show that the system of equations $x+2y+z=11$, $4x+6y+5z=8$, $2x+2y+3z=19$ is inconsistent.

Or

(b) Find the rank of the matrix $\begin{pmatrix} 0 & 1 & 2 & 1 \\ 2 & -3 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$.

15. (a) Test whether characteristic root of Hermitian matrix can be a complex number.

Or

(b) Prove that $f(x, y) = x_1 y_2 - x_2 y_1$ is a bilinear form on $V_2(R)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that the intersection of two subspaces of a vector space is also a subspace. Also check whether the union of two subspaces is a subspace.

Or

(b) Let V and W be vector spaces over a field F and $T: V \to W$ be an epimorphism. Then prove :

(i) $Ker\ T = V_1$ is a subspace of V

(ii) $\frac{V}{V_1} \cong W$.

17. (a) Let V be a finite dimensional vector space over a field F. Let W be a subspace of V. Then prove :

(i) $\dim W \le \dim V$

(ii) $\dim \frac{V}{W} = \dim V - \dim W$.

Or

(b) Show that any two bases of a finite dimensional vector space V have the same number of elements.

18. (a) Explain the Gram – Schmidt orthogonalization process.

Or

(b) Let V be a finite dimensional inner product space. Let W be a subspace of V. Then show that $V = W \oplus W^{\perp}$.

19. (a) Verify Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

Or

(b) Verify whether the following system of equations is consistent. If it is consistent, solve it : $2x - y + 3z = 8$; $x - 2y - z = -4$; $3x + y - 4z = 0$.

20. (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

Or

(b) Reduce the quadratic form $2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.

(6 pages) **Reg. No. :** ....

Code No. : 30423 E **Sub. Code : CMMA 52**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics — Core

REAL ANALYSIS

(For those who joined in July 2021-2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The diameter of any non-empty subset in a discrete metric space is ————

   (a) 0

   (b) ∞

   (c) $-\infty$

   (d) 1

2. In R with usual metric, for $a \in R$, the open ball $B(a, r) =$

   (a) $(a-r, a+r)$ (b) $[a-r, a+r]$

   (c) $(a+r, a-r]$ (d) $[a+r, a+r)$

3. The set of irrational number in R is

   (a) Open (b) Closed

   (c) Complete (d) Dense

4. The incorrect statement is

   (a) R with usual metric is complete

   (b) C with usual metric is complete

   (c) Q with usual metric is complete

   (d) Discrete metric is complete

5. The set of discontinuities of the function $f : R \to R$ defined by $f(x) = [x]$, the integral part of x, is ————

   (a) N (b) $\{0\}$

   (c) ϕ (d) Z

6. Which of the following is a dense set in R with discrete metric?

   (a) R (b) Q

   (c) $R-Q$ (d) Z

7. Which of the following is not connected subset of R?

(a) $[1,2]$ (b) $(0, \infty)$

(c) $(3, 4)$ (d) $(1, 2) \cup (2, 3)$

8. In a discrete metric space, the only connected subsets are

(a) Finite sets (b) The whole set

(c) Singleton set (d) All proper subsets

9. In a discrete metric space the only compact subsets are

(a) Finite sets

(b) Singleton sets

(c) The whole set

(d) Any proper subsets

10. Which of the following is equivalent to compactness in a metric space M?

(a) M is totally bounded

(b) M is complete

(c) Every bounded subset of M has a limit point

(d) Every infinite subset of M has a limit point

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Determine whether $d(x, y)$ defined on R by $d(x, y) = (x - y)^2$ is a metric or not.

Or

(b) Prove that in any metric space the intersection of a finite number of open sets is open.

12. (a) Prove that in any metric space, arbitrary intersection of closed sets is closed.

Or

(b) Prove that let (M, d) be a metric space. Then any convergent sequence in M is a Cauchy sequence.

13. (a) Let f, g be continuous real valued functions on a metric space M. Let $A = \{x : x \in M \text{ and } f(x) < g(x)\}$. Prove that A is open.

Or

(b) Prove that $f : [0, 1] \to R$ defined by $f(x) = x^2$ is uniformly continuous on $[0, 1]$.

14. (a) Prove that a metric space M is connected iff there does not exist a continuous function f from Monto the discrete metric space $\{0, 1\}$.

Or

(b) If A and B are connected subsets of a metric space M and if $A \cap B \neq \phi$, prove that $A \cup B$ is connected.

15. (a) Prove that any compact subset A of a metric space M is bounded.

Or

(b) Prove that let A be a subset of a metric space M. If A is totally bounded then A is bounded.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Let (m, d) be a metric space. Define $d_1(x, y) = \min\{1, d(x, y)\}$ prove that d_1 is a metric on M.

Or

(b) Let (M, d) be a metric space. Define $\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ prove that d and ρ are equivalent metric S on M.

17. (a) C with usual metric is complete – Prove that.

Or

(b) State and prove Baire's category theorem.

18. (a) Prove that f is continuous iff inverse image of every open set is open.

Or

(b) Prove that the metric spaces $(0, 1)$ and $(0, \infty)$ with usual metrics are homeomorphic.

19. (a) Prove that a subspace of R is connected iff it is an interval.

Or

(b) State and prove the intermediate value theorem.

20. (a) State and prove Heine Borel Theorem.

Or

(b) Prove that a metric space M is compact iff any family of closed sets with finite intersection property has non-empty intersection.

———————

(8 pages) **Reg. No. :** ....

Code No. : 30424 E **Sub. Code : CMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics — Core

STATICS

(For those who joined in July 2021- & 2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If two forces P and Q are at right angles to each other, then the resultant force is ————

   (a) $P+Q$ (b) $\sqrt{P^2+Q^2}$

   (c) $\sqrt{P^2-Q^2}$ (d) $P-Q$

2. A force has no resolved part in its ———— direction.

   (a) parallel (b) perpendicular

   (c) own (d) all

3. A couple is the effect of two ———— forces.

   (a) like parallel

   (b) coplanar

   (c) unlike parallel

   (d) unlike, equal parallel

4. If the line of action of a force passes through the point O, then the moment of that force about the point is ————

   (a) positive (b) negative

   (c) zero (d) not defined

5. Three coplanar forces P, Q, R are in equilibrium and are parallel. Which of the following is true?

   (a) P and Q from a couple

   (b) P, Q, R are like parallel forces only

   (c) One must be proportional to distance between other two

   (d) Atleast one must be in opposite direction of other two

6. Let P, Q and R be coplanar forces acting on a rigid body. If it is in equilibrium, which of the following cannot be true?

(a) P and Q – like parallel

(b) P and Q – unlike parallel

(c) P and Q – couple forces

(d) P, Q, R – like parallel

7. The maximum value of function is ————

(a) μ/R (b) μ

(c) μR (d) R/μ

8. When one body in contact with another is in equilibrium, then the friction exerted is called ————

(a) limiting friction (b) statical friction

(c) dynamical friction (d) passive friction

9. The intrinsic equation of the catenary is ————

(a) $s = c\tan\psi$ (b) $s = c\sin\psi$

(c) $s = c\cos\psi$ (d) $s = c\sec\psi$

10. Which of the following is true in a catenary?

(a) $s^2 = y^2 + c^2$ (b) $y^2 = c^2 + s^2$

(c) $y^2 = s^2 - c^2$ (d) $y^2 = c^2 - s^2$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Lami's theorem.

Or

(b) $ABCDEF$ is a regular hexagon and at A, act forces represented by $\overline{AB}, 2\overline{AC}, 3\overline{AD}, 4\overline{AE}$ and $5\overline{AF}$. Show that the magnitude of the resultant is $AB.\sqrt{351}$ and that it makes an angle $\tan^{-1}\left(\frac{7}{\sqrt{3}}\right)$ with AB.

12. (a) A uniform circular plate is supported horizontally at three points A, B, C of its circumference. Show that the pressure on the supports in the ratio $\sin 2A : \sin 2B : \sin 2C$.

Or

(b) Find the resultant of two unlike and unequal parallel forces acting on a rigid body.

13. (a) A uniform sold hemisphere of weight W rests with its curved surface on a smooth horizontal plane. A weight w is suspended from a point on the rim of the hemisphere. If the plane base of the rim is inclined to the horizontal at an angle θ, prove that $\tan\theta = \frac{8w}{3W}$.

Or

(b) A uniform beam of length l and weight W hangs from a fixed point by two strings of lengths a and b. Prove that the inclination of the rod to the horizon is $\sin^{-1}\left(\frac{a^2-b^2}{l\sqrt{2(a^2+b^2)-l^2}}\right)$.

14. (a) A particle of weight 30 Kgs resting on a rough horizontal plane is just on the point of motion when acted on by horizontal forces of 6 Kg wt and 8 kg wt at right angles to each other. Find the coefficient of friction between the particle and the plane and the direction in which the friction acts.

Or

(b) Two particles P and Q each of weight W on two equally rough inclined planes CA and CB of the same height, placed back to back are connected by a light string which passes over the smooth top edge C of the planes. Show that if the particles are on the point of slipping, the difference of the inclinations of the planes is double the angle of friction.

15. (a) The span of a suspension bridge is 100 metres and the sag at the middle of the chain is 10 metres, if the total load on each chain is 750 quintals, find the greatest tension in each chain and tension at the lowest point.

Or

(b) Describe the geometrical properties of the common catenary.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Two beads of weight w and w^{-1} cam slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle 2β at the centre of circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination of the string to the horizontal is given by $\tan\alpha = \frac{w-w'}{w+w'}\tan\beta$.

Or

(b) E is the middle point of the side CD of a square ABCD. Force $16, 20, 4\sqrt{5}, 12\sqrt{2}$ Kg.wt. act along AB, AD, EA, CA in the directions indicated by the order of the letters. Show that they are in equilibrium.

17. (a) Forces P, Q, R act along the sides BC, AC, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = 1/2P$.

Or

(b) State and prove Varigon's theorem on moments.

18. (a) A heavy uniform rod of length 2a, rests partly within and partly without a smooth hemispherical bowl of radius r, fixed with its rim horizontal. If α is the inclination of the rod to the horizon, show that $2r\cos 2\alpha = a\cos\alpha$. Also calculate the reactions at the extremities show also that the length of the rod should lie between $4r$ and $2r\sqrt{2/3}$ in order that it may rest in equilibrium thus.

Or

(b) A uniform rod of length $2l$ rests with its lower end in contact with a smooth vertical wall. It is supported by a string of length a, one end of which is fastened to a point in the wall and the other end to a point in the rod at a distance b from its lower end. If the inclination of the string to the vertical be θ, show that $\cos^2\theta = \dfrac{b^2(a^2 - b^2)}{a^2 l(2b - l)}$.

19. (a) Find the conditions for equilibrium of a body on a rough inclined plane under a force parallel to the plane.

Or

(b) A weight can be supported on a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally. Show that the weight is $\dfrac{PQ}{\sqrt{Q^2\sec^2\lambda - P^2}}$ where λ is the angle of friction.

20. (a) A telegraph wire stretched between two poles at a distance 'a' feet apart sags n feet in the middle. Prove that the tension at the ends is approximately $w\left(\dfrac{a^2}{8n} + \dfrac{7n}{6}\right)$ where w is the weight of unit length of wire.

Or

(b) Derive the equation of the common catenary.

(7 pages) **Reg. No. :** ....

Code No. : 30425 E **Sub. Code : CMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics – Core

INTEGRAL TRANSFORMS AND Z TRANSFORMS

(For those who joined in July 2021&2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. $F(f(x-a))=$ ————————.

(a) $F\left(\frac{s}{a}\right)$ (b) $e^{ias}F(s)$

(c) $e^{-ias}F(s)$ (d) $F(as)$

2. $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt =$ ————————.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) π

3. $f(x)=e^{-ax}$, $a>0$ then $F_c(f(x))=$ ————————.

(a) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (b) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

(c) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2-a^2}$ (d) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2-a^2}$

4. $F_s\left(\frac{x}{x^2+1}\right)=$ ————————.

(a) $\sqrt{\frac{2}{\pi}}e^{-s}$ (b) $\sqrt{\frac{2}{\pi}}e^{s}$

(c) $\sqrt{\frac{\pi}{2}}e^{-s}$ (d) $\sqrt{\frac{\pi}{2}}e^{s}$

5. In $(0, l)$, the finite Fourier cosine transform of $f(x)$ is $\bar{f}_c(n)=$ ————————.

(a) $\int_0^l f(x)\cos n\pi x\, dx$ (b) $\int_0^l f(x)\cos nx\, dx$

(c) $\int_0^\infty f(x)\cos\frac{n\pi x}{l}dx$ (d) $\int_0^l f(x)\cos\frac{n\pi x}{l}dx$

6. In $(0, l)$, $F_S(f'(x)) =$ ____________.

(a) $\frac{n\pi}{l}\bar{f}_c(n)$ (b) $\frac{n\pi}{l}\bar{f}_S(n)$

(c) $-\frac{n\pi}{l}\bar{f}_S(n)$ (d) $-\frac{n\pi}{l}\bar{f}_c(n)$

7. $Z(1) =$ ____________.

(a) $\frac{1}{z-1}$ (b) $\frac{z}{z-1}$

(c) $\frac{1}{1-z}$ (d) $\frac{z}{1-z}$

8. $Z\left(\sin\frac{n\pi}{2}\right) =$ ____________.

(a) $\frac{z}{z+1}$ (b) $\frac{z}{z-1}$

(c) $\frac{z}{z^2-1}$ (d) $\frac{z}{z^2+1}$

9. $Z^{-1}\left(\frac{z}{z-2}\right) =$ ____________.

(a) 2^n (b) e^{-2n}

(c) $\cos\frac{n\pi}{2}$ (d) $\sin\frac{n\pi}{2}$

10. $Z^{-1}\left(e^{1/z}\right) =$ ____________.

(a) $n!$ (b) $(n-1)!$

(c) $\frac{1}{n!}$ (d) $\frac{1}{(n-1)!}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove Parseval's identity.

Or

(b) State and prove convolution theorem for Fourier transform.

12. (a) Verify : $F_c(x) = -\sqrt{\frac{2}{\pi}}\frac{1}{s^2}$.

Or

(b) Find : $F_S\left(\frac{e^{-ax}}{x}\right)$.

13. (a) In $(0, l)$, evaluate $F_S\left(e^{ax}\right)$ and $F_C\left(e^{ax}\right)$.

Or

(b) Find $f(x)$ if $\bar{f}_c(n) = \frac{1-\cos n\pi}{n^2\pi^2}$ in $(0, \pi)$.

14. (a) Estimate $Z\left(\frac{1}{n}\right)$ if $|z|>1,\ n>0$.

Or

(b) Prove :

(i) $Z(\delta(n-k))=\frac{1}{z^k}$

(ii) $Z(u(n-1))=\frac{1}{z-1}$

15. (a) Compute $Z^{-1}\left(\frac{Z}{(Z-1)^2(Z+1)}\right)$.

Or

(b) Find $4^n * 3^n$ and hence estimate $Z^{-1}\left(\frac{Z^2}{(Z-4)(Z-3)}\right)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Verify whether the function $e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transform.

Or

(b) Obtain the Fourier transform of the function $f(x)=\begin{cases} x, & |x|<a \\ 0, & |x|>a \end{cases}$.

17. (a) Prove : $F_C\left(\frac{1}{x^2+a^2}\right)=\sqrt{\frac{\pi}{2}}\frac{e^{-as}}{a}$.

Or

(b) Evaluate : $\int_0^\infty \frac{dx}{(x^2+4)(x^2+1)}$.

18. (a) In $(0,\pi)$, find $F_C(f(x))$ and $F_S(f(x))$ if $f(x)=\begin{cases} 1, & 0<x<\frac{\pi}{2} \\ -1, & \frac{\pi}{2}<x<\pi \end{cases}$.

Or

(b) Find $F_s(\cos ax)$ and $F_c(\sin ax)$ in $(0,\pi)$.

19. (a) State and prove find value theorem for Z-transformation.

Or

(b) Evaluate $Z((n-1)a^{n-1})$.

20. (a) Solve : $y_{n+2} + y_n = 2,\ y_0 = y_1 = 0$

Or

(b) Find $Z^{-1}\left(\dfrac{z^3 - 20z}{(z-2)^3(z-4)}\right)$.

(6 pages)

Reg. No. : ....

Code No. : 30443 E **Sub. Code : CEMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics

Major Elective — COMBINATORIAL MATHEMATICS

(For those who joined in July 2021-2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of P(3) is

(a) 3 (b) 3!

(c) 5 (d) (3-1)!

2. The value of $c(n,1)$

(a) 1 (b) n

(c) $n-1$ (d) 0

3. If $n=2$, the total number of different pairing is

(a) 15 (b) 6

(c) 1 (d) 3

4. How many number of vertices needs for perfect matching?

(a) odd (b) even

(c) both (a) and (b) (d) none of these

5. Which one of the following is derangement of 12345?

(a) 23514 (b) 13254

(c) 23541 (d) 54321

6. Number of partition of 3 is

(a) 3 (b) 5

(c) 7 (d) 2

7. If $A=\{1,2,3,4\}$, $B=\{3,4,5,6\}$, then $|A \cup B|$ is

(a) 6 (b) 4

(c) 2 (d) 3

8. In 4×4 board, the value of $r_4(C)$ is

(a) 96 (b) 24

(c) 16 (d) 1

9. For a (b, v, r, k, λ) – configuration, $b \geq v$. This result is proved by

(a) Riemann (b) Fisher

(c) Murphy (d) Rook

10. If J is $v \times v$, then $J^2 =$

(a) J (b) vJ

(c) $v^2 J$ (d) vJ^2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the unordered selections.

Or

(b) Prove that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$.

12. (a) Explain pairings within a set.

Or

(b) Write a short note on Latin square.

13. (a) Explain the problem of derangements.

Or

(b) Solve $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ given that $a_1 = 2, a_2 = 6, a_3 = 20$.

14. (a) Explain the inclusion – exclusion principle.

Or

(b) Find the rook polynomial for an ordinary 4 × 4 board.

15. (a) Explain the square block designs.

Or

(b) Explain the error-correcting codes.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove binomial theorem.

Or

(b) Expand $(1+x)^8$ and $(1-x)^8$.

17. (a) State and prove marriage theorem.

Or

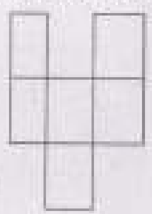
(b) If a graph has $2n$ vertices each of degree $\geq n$, then prove that the graph has a perfect matching.

18. (a) Prove that $a_n = n!\left\{1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots(-1)\frac{1}{n!}\right\}$.

Or

(b) Solve $a_n = 6a_{n-1} - 9a_{n-2}$, given that $a_0 = 2, a_1 = 6$ unity $f(x) = \sum_{n=0}^{\infty} a_n x^n$ and showing that $f(x) = 2(1-3x)^{-1}$.

19. (a) Find the Rook polynomial of the board.



Or

(b) How many permutations one there of the digits 1, 2,...8 include none of the patterns 12, 34, 56, 78 appears?

20. (a) Prove that there is no finite projective plane of order 6.

Or

(b) State and prove Fisher's theorem.

(8 pages) **Reg. No. :** ....

Code No. : 30444 E **Sub. Code : CEMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

Fifth Semester

Mathematics — Major Elective

OPERATIONS RESEARCH — I

(For those who joined in July 2021-2022 only)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If the constraints of an LPP is ≤ type, then the variable to be added is

(a) slack (b) surplus

(c) artificial (d) none

2. Any solution to a general LPP which satisfies non negative restrictions of the problem is called a ———— to the general LPP.

(a) solution (b) feasible solution

(c) optimum solution (d) negative solution

3. The cost of surplus variable is

(a) 1 (b) –1

(c) 0 (d) M

4. The Big-M method is also known as

(a) Penalty method (b) Simplex method

(c) Both (a) and (b) (d) None of the above

5. In a transportation problem feasible solution exists when the number of basic cell is

(a) $m+n-1$ (b) $m+n+1$

(c) $m-n-1$ (d) $m-n+1$

6. A transportation problem is balanced if

(a) $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ (b) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

(c) $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$ (d) $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

7. The optimum assignment is

(a) $N = n$ (b) $N \neq n$

(c) $N < n$ (d) $N > n$

8. The algorithm to solve the assignment problem was developed by

(a) D konig (b) Charnes

(c) F.L. Hitchcock (d) Kuhn

9. The assignment problem is balanced if

(a) the number of rows and columns are equal

(b) the number of rows and columns are not equal

(c) number of rows is less than the number of columns

(d) number of rows is greater than the number of columns

10. In a sequencing algorithm, no passing rule is

(a) the order of the first and last job

(b) order in which the machines complete the job

(c) the same order of jobs is maintained over each machine

(d) none

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Solve graphically :

Maximize : $z = 3x_1 + 2x_2$

Subject to :

$$x_1 - x_2 \le 1$$
$$x_1 + x_2 \ge 3$$
$$x_1, x_2 \ge 0.$$

Or

(b) Define slack and surplus variables.

12. (a) Write the dual of

Maximize $z = 2x_1 + x_2$

Subject to

$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$
$$x_1, x_2 \ge 0.$$

Or

(b) Use dual simplex method to solve the following LPP

Minimize $z = 3x_1 + x_2$

Subject to the constraints

$$x_1 + x_2 \ge 1$$
$$2x_1 + 3x_2 \ge 2$$
$$x_1 \ge 0, x_2 \ge 0.$$

13. (a) Obtain an initial basic feasible solution to the following transportation problem by North – West corner rule.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Demand	200	225	275	250	

Or

(b) Explain Vogel's approximation method in transportation problem.

18. (a) Explain the MODI method in transportation problem.

Or

(b) Solve the transportation problem and find the optimum solution.

	1	2	3	4	Availability
A	10	8	11	7	20
B	9	12	14	6	40
C	8	9	12	10	35
Demand	16	18	31	30	

19. (a) Explain Hungarian method in assignment problem.

Or

(b) Solve the following assignment problem

	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

20. (a) Solve the following sequencing problem

Machines

Jobs	M_1	M_2	M_3	M_4
A	13	8	7	14
B	12	6	8	19
C	9	7	5	15
D	8	5	6	15

Or

(b) Solve the given sequencing problem in the order ABC

Jobs :	1	2	3	4	5	6
Machine A :	12	10	9	14	7	9
Machine B :	7	6	6	5	4	4
Machine C :	6	5	6	4	2	4

14. (a) Write the mathematical formulation of an assignment problem.

Or

(b) Solve the following assignment problem

	1	2	3
A	120	100	80
B	80	90	110
C	110	140	120

15. (a) Solve the following sequencing problem :

Job :	1	2	3	4	5	6	7
M_1 :	3	12	15	6	10	11	9
M_2 :	8	10	10	6	12	1	3

Or

(b) Explain the procedure for solving n jobs and 2 machines in sequencing problem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Use simplex method to solve the following LPP

Maximize $z = x_1 + x_2 + 3x_3$

Subject to

$$3x_1 + 2x_2 + x_3 \le 3$$
$$2x_1 + x_2 + 2x_3 \le 2$$
$$x_1, x_2, x_3 \ge 0.$$

Or

(b) Explain simplex method.

17. (a) Use Penalty method to solve

Maximize $z = 3x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 + x_2 + x_3 \le 2$$
$$3x_1 + 4x_2 + 2x_3 \ge 8$$
$$x_1, x_2, x_3 \ge 0.$$

Or

(b) Solve the following LPP using dual simplex method.

Minimize $z = 2x_1 + 3x_2$

Subject to

$$2x_1 + 3x_2 \le 30$$
$$x_1 + 2x_2 \ge 10$$
$$x_1, x_2 \ge 0.$$